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COMMENT

Comment on 'Critical behaviour of spin-s Heisenberg antiferromagnetic chains: analytic and numerical results'

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Abstract. We make some observations on a recent paper by Affleck *et al* about the singlet excitation in the spin-$s=1$ integrable model and logarithmic corrections for integrable spin-s models.

In a recent paper Affleck *et al* [1] studied the critical behaviour of the various Hamiltonians with spin $s=\frac{1}{2}, 1, \frac{3}{2}$ including the logarithmic corrections for gaps and correlation functions. In section 6 of [1] Affleck *et al* analysed the excitations in the exactly integrable Hamiltonians with arbitrary spins s. They claimed that the singlet state for the spin-$s=1$ Babudjan–Takhtajan model [2, 3] is characterised by $L/2-1$ (L is the lattice size) pairs of roots $x+\mathrm{i}y$; $|y|=\frac{1}{2}+\varepsilon$, $\varepsilon<0$ plus a pair at exactly $\pm\mathrm{i}$, and they mentioned that this result is different from that found by Takhtajan [2]. This picture proposed by Affleck *et al* is not correct. It is easy to see from Bethe ansatz equations [2, 3] that $\pm\mathrm{i}$ is the solution if one puts another zero in the origin, forming an exact 3-string. The correct singlet excited state for a spin-1 integrable Hamiltonian is characterised by $L/2-2$ pairs of roots $x+\mathrm{i}y$, $|y|=\frac{1}{2}+\varepsilon$, $\varepsilon<0$ plus an exact 3-string and a 1-string $\{\pm\mathrm{i}, 0; 0\}$ in the origin. This result and analysis for spin $s\geqslant\frac{3}{2}$ was obtained before [4], by comparing the solution of Bethe ansatz equations with an exact diagonalisation of an associated Hamiltonian. This last result is in perfect agreement with Takhtajan's picture for the spin-$s=1$ singlet state.

Our final observations concern the logarithmic corrections. In section 5 of [1] the authors introduced a frustrating second-nearest-neighbour term in the $s=\frac{1}{2}$ isotropic Heisenberg model in order to vary the marginal coupling constant, responsible for the logarithmic corrections. There is another, elegant, way to do that, introducing an anisotropy γ ($0\leqslant\gamma\leqslant\pi/2s$) in these integrable spin-s models [4]. In this case the integrability is preserved and we have a marginal operator responsible for the continuous variation of the critical exponents with γ [5, 6]. For $\gamma\neq 0$ corrections to scaling for the ground state $E(L)_0$ have the form [5, 6]:

$$\frac{E(L)_0}{L}-l_\infty=-\frac{\pi^2\sin(2s\gamma)}{24s\gamma L^2}\left(c+\mathrm{O}(L^{-2})+\mathrm{O}(L^{-4\gamma/(\pi-2s\gamma)})+\ldots\right) \qquad (1)$$

where l_{00} is the bulk limit of the ground-state energy and $c=3s/(s+1)$.

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The corrections $O(L^{-2})$ and $O(L^{-4\gamma/(\pi-2s\gamma)})$ in (1) are consequences of irrelevant operators with conformal dimensions $x_1=4$; and $x_2=\pi/s(\pi-2s\gamma)+2s-1/s$, respectively. When $\gamma\to 0$, the conformal dimension of the operator responsible for the corrections to scaling, $x_2\to 2$. Strictly at $\gamma=0$, an infinite number of terms become equally important in (1), and the original corrections renormalise, giving rise to logarithmic corrections [7].

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References

[1] Affleck I, Gepner D, Schulz H J and Ziman T 1989 *J. Phys. A: Math. Gen.* **22** 511
[2] Takhtajan L 1982 *Phys. Lett.* **87A** 479
[3] Babudjan J 1983 *Nucl. Phys.* B **215** 317
[4] Alcaraz F C and Martins M J 1988 *J. Phys. A: Math. Gen.* **21** 4397; 1989 *J. Phys. A: Math. Gen.* **22** L99
[5] Sogo K 1984 *Phys. Lett.* **104A** 51
[6] Alcaraz F C and Martins M J 1988 *Phys. Rev. Lett.* **61** 1529; 1989 *J. Phys. A: Math. Gen.* **22** 1829
[7] Cardy J L 1986 *J. Phys. A: Math. Gen.* **19** L1093